Aljabar Boolean (Sistem Digital A – Narendro Arifia)

Boolean Postulates in 0 and 1

OR	AND	NOT
$0 + 0 = 0$	$0 \cdot 0 = 0$	$\overline{0} = 1$
$0 + 1 = 1$	$0 \cdot 1 = 0$	$\overline{1} = 0$
$1 + 0 = 1$	$1 \cdot 0 = 0$	
$1 + 1 = 1$	$1 \cdot 1 = 1$	

Boolean Theorems in One Variable

OR	AND	NOT
$A + 0 = A$	$A \cdot 0 = 0$	$\overline{\overline{A}} = A$
$A + 1 = 1$	$A \cdot 1 = A$	
$A + A = A$	$A \cdot A = A$	
$A + \overline{A} = 1$	$A \cdot \overline{A} = 0$	

Boolean Theorems in More Than One Variable

Commutation rules:

$A + B = B + A$

$A \cdot B = B \cdot A$

Absorption rules:

$A + (A \cdot B) = A$

$A \cdot (A + B) = A$

Association rules:

$A + (B + C) = (A + B) + C$

$A \cdot (B \cdot C) = (A \cdot B) \cdot C$

Distribution rules:

$A \cdot (B + C) = (A \cdot B) + (A \cdot C)$

$A + (B \cdot C) = (A + B) \cdot (A + C)$

DeMorgan's theorems:

$\overline{A + B} = \overline{A} \cdot \overline{B}$

$\overline{A \cdot B} = \overline{A} + \overline{B}$

Teori De Morgan

Pernyataan minterm $\overline{\overline{C} \cdot \overline{B} \cdot \overline{A} + C \cdot B \cdot \overline{A}} = Y$

pertama $\overline{(\overline{C} + \overline{B} + \overline{A}) \cdot (C + B + \overline{A})}$

kedua $\overline{(\overline{\overline{C}} + \overline{\overline{B}} + \overline{\overline{A}}) \cdot (\overline{C} + \overline{B} + \overline{\overline{A}})}$

ketiga $\overline{\overline{(\overline{\overline{C}} + \overline{\overline{B}} + \overline{\overline{C}}) \cdot (\overline{C} + \overline{B} + \overline{\overline{A}})}}$

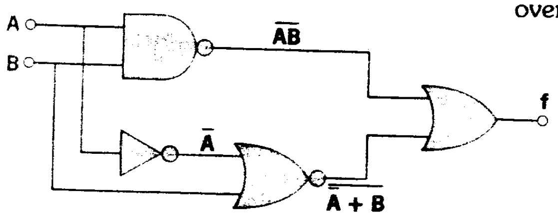
keempat Hilangkan strip di atas yang berjumlah genap

Pernyataan maksterm $(C + B + A) \cdot (\overline{C} + \overline{B} + A) = Y$

Pernyataan maksterm	$(C + B + A)\cdot(\overline{C} + \overline{B} + A) = Y$
pertama	$(C \cdot B \cdot A) + (\overline{C} \cdot \overline{B} \cdot A)$
kedua	$(\overline{C} \cdot \overline{B} \cdot \overline{A}) + (\overline{\overline{C}} \cdot \overline{\overline{B}} \cdot \overline{A})$
ketiga	$\overline{(\overline{C} \cdot \overline{B} \cdot \overline{A}) + (\overline{\overline{C}} \cdot \overline{\overline{B}} \cdot \overline{A})}$
keempat	Hilangkan strip di atas yang berjumlah genap
Pernyataan minterm	$\overline{\overline{C} \cdot \overline{B} \cdot \overline{A} + C \cdot B \cdot \overline{A}} = Y$

Contoh 1

Analyze the logic circuit



The suboutputs are noted on the diagram. The overall function can be simplified as follows:

$$f = \overline{AB} + \overline{\overline{A} + B}$$

$$= (\overline{A} + \overline{B}) + A\overline{B} \quad \text{(DeMorgan's rule)}$$

$$= \overline{A} + \overline{B}(1 + A) \quad \text{(Distribution)}$$

$$= \overline{A} + \overline{B} \quad (1 + A = 1)$$

$$= \overline{AB} \quad \text{(DeMorgan's rule)}$$

A	B	$\overline{AB}$	$\overline{\overline{A} + B}$	f
0	0	1	0	1
0	1	1	0	1
1	0	1	1	1
1	1	0	0	0

Contoh 2

$$\bar{A}B + A\bar{B} = \overline{(A + \bar{B})(\bar{A} + B)} \qquad \text{(DeMorgan's theorems)}$$

$$= \overline{A\bar{A} + AB + \bar{A}\bar{B} + B\bar{B}} \qquad \text{(Multiplication)}$$

$$= \overline{\bar{A}\bar{B} + AB} \qquad (A\bar{A} = 0 \text{ and } B\bar{B} = 0)$$

$$= (A + B)(\bar{A} + \bar{B}) \qquad \text{(DeMorgan's theorems)}$$

$$= (A + B)\overline{AB} \qquad \text{(De Morgan's theorems)}$$

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 A =  1 1 0 0
+B =  1 0 1 0
     10 1 1 0
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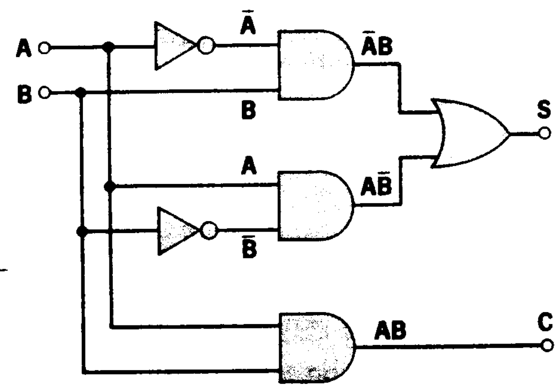
(a) Addition

A	B	S	C
0	0	0	0
0	1	1	0
1	0	1	0
1	1	0	1

(b) Truth table

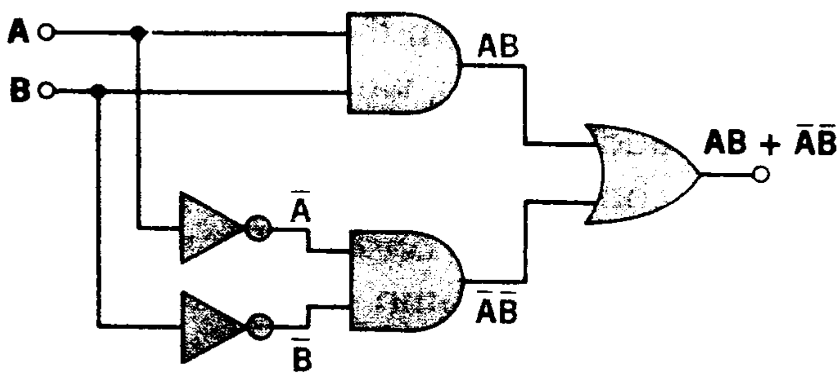


(c) Half-adder circuit

Contoh 3 :

$$\overline{\bar{A}B + A\bar{B}} = (A + \bar{B})(\bar{A} + B) = AB + \bar{A}\bar{B}$$

This is an equality comparator in that the output is 1 if A and B are equal. This function is highly useful in digital computer operation. Straightforward synthesis results in the circuit



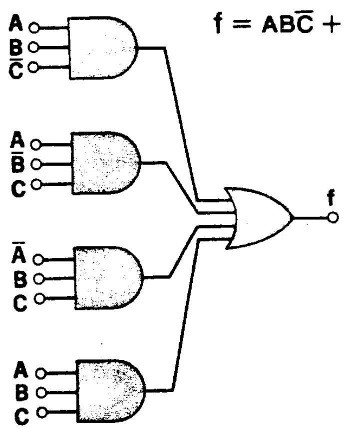
An equality comparator.

Contoh 4:

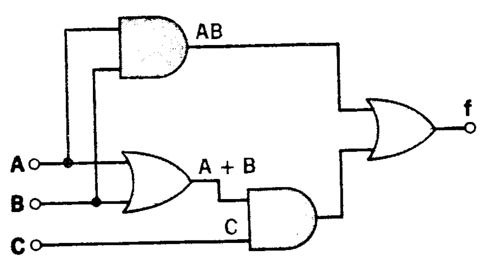
$$f = AB\bar{C} + A\bar{B}C + \bar{A}BC + ABC$$

A	B	C	f
0	0	0	0
0	0	1	0
0	1	0	0
0	1	1	1
1	0	0	0
1	0	1	1
1	1	0	1
1	1	1	1

(a) Truth table



(b) Circuit realization



(c) After Boolean simplification

since ABC + ABC = ABC. Factoring by the distribution rule yields

$$f = AB(\bar{C} + C) + C(AB + A\bar{B} + \bar{A}B)$$

Since $\bar{C} + C = 1$ and $AB + A\bar{B} + \bar{A}B = A + B$, the function becomes

$$f = AB + C(A + B)$$